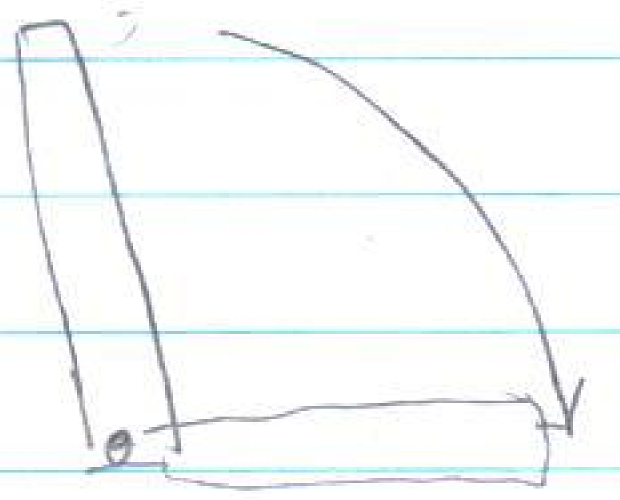
Review Prob

A rod, Length L, mass M P61, Ch 10



a) Find its angular speed

$$\cancel{W_{ext}} = \Delta KE + \Delta PE$$

$$\cancel{KE_i} + PE_i = KE_f + \cancel{PE_f}$$

$$Mg\frac{L}{2} = \frac{1}{2} I \omega_f^2 \qquad I = \frac{1}{3} ML^2$$

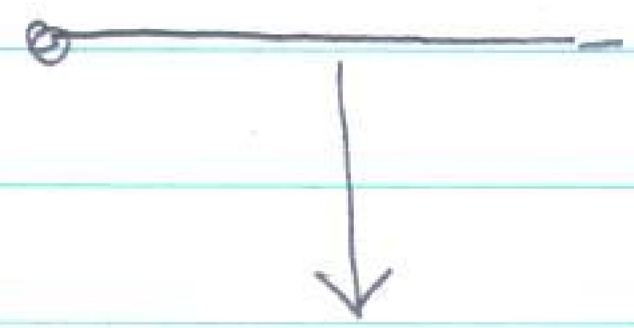
$$\cancel{M}g\frac{L}{\cancel{2}} = \frac{1}{\cancel{2}} \frac{1}{3} ML^2 \omega_f^2$$

$$\sqrt{\frac{3g}{L}} = \omega_f$$

b) Find the angular acceleration when horizontal

c) Find the x and y components of the $\vec{a}_{cm}$

d) Determine the force on the pin



b) $\vec{\tau} = I\vec{\alpha}$

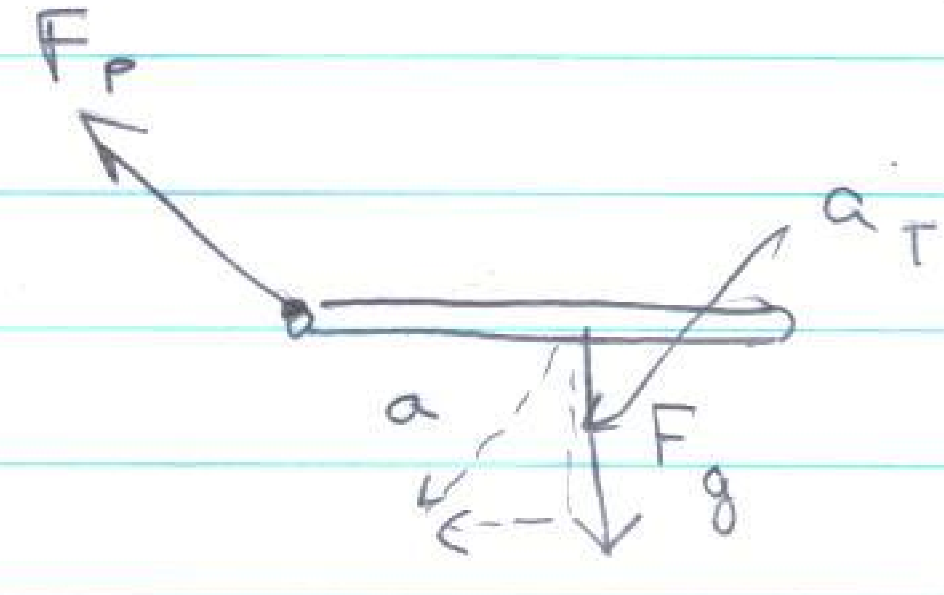
$\frac{L}{2}\cancel{M}g\ -\hat{k} = \frac{1}{3}\cancel{M}L^2\ \vec{\alpha}$

$\frac{3}{2}\frac{g}{L}(-\hat{k}) = \vec{\alpha}$ $\qquad |\vec{\alpha}| = \alpha = \frac{3}{2}\frac{g}{L}$ clockwise

c) $a^x_{cm} = a_c = \omega_r^2 r_{cm} = \frac{3g}{L} \cdot \frac{L}{2} = \frac{3}{2}g$ in

$a^y_{cm} = a_t = \alpha\ r_{cm} = \frac{3}{2}\frac{g}{L} \cdot \frac{L}{2} = \frac{3}{4}g$ down

d) $\sum \vec{F} = m\vec{a}$



$\sum F^x = ma^x$

$F_p^x = -m\,a_c$

$F_p^x = -m\frac{3}{2}g \leftarrow$ in

$\sum F^y = ma^y$

$-mg + F_p^y = m\left(-\frac{3}{4}g\right) \Rightarrow F_p^y = \frac{1}{4}mg$

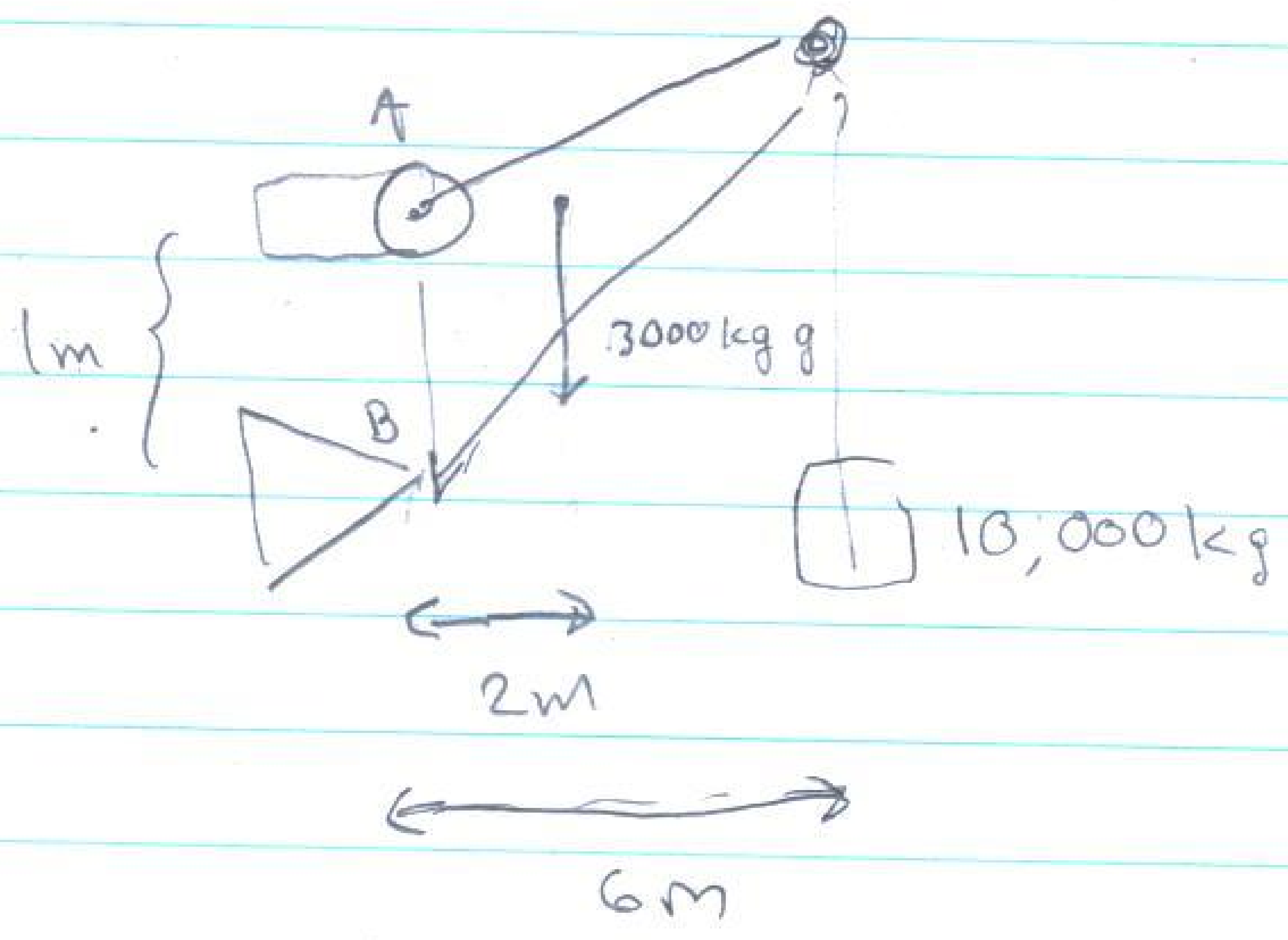
What we have labelled F_p is the force on board by pin

want $\vec{F}_{on\ pin} = -\vec{F}_{board}$

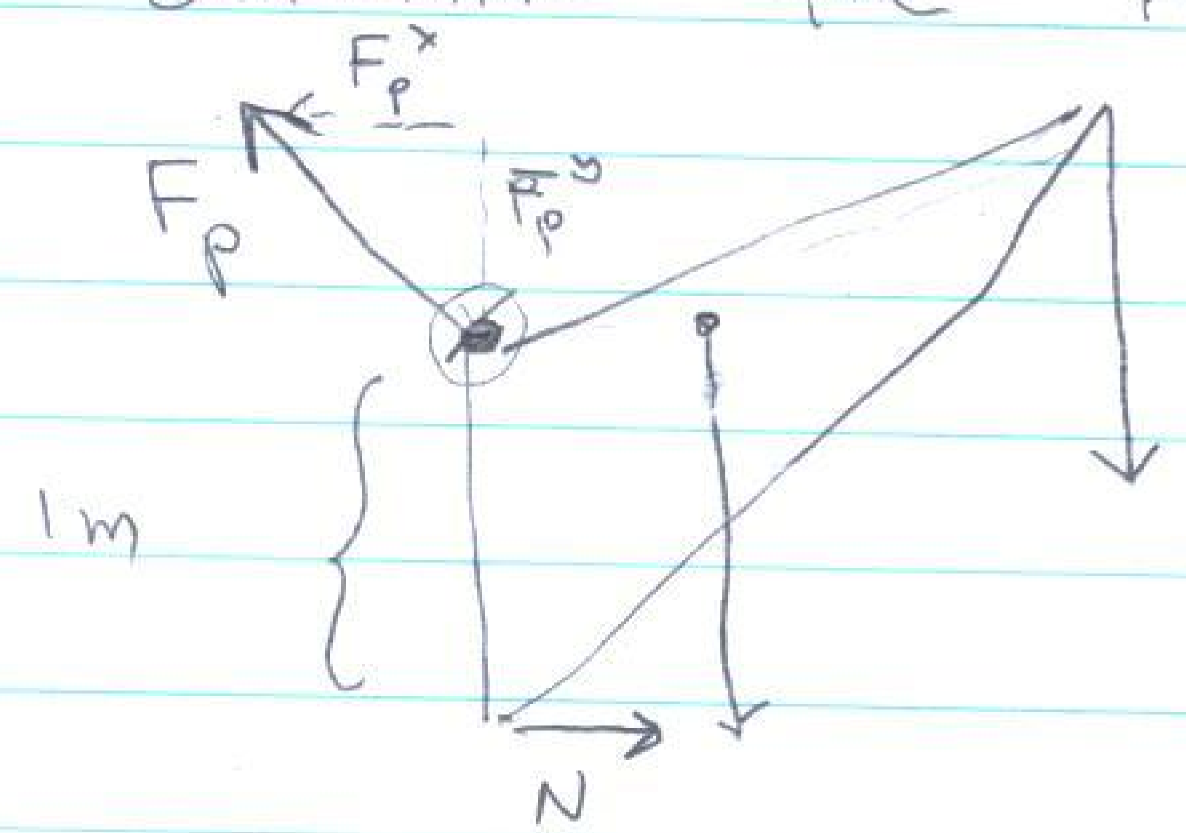
$$\vec{F}_{on\ pin} = m\frac{3}{2}g\,\hat{\imath} - \frac{1}{4}mg\,\hat{\jmath}$$

$$|F_p| = mg\left(\left(\frac{3}{2}\right)^2 + \left(\frac{1}{4}\right)^2\right)^{\frac{1}{2}} \quad \text{etc}$$

Review Prob P47 Ch 12 book



Determine the reaction forces at A & B



$\sum F^x = 0$ $\qquad\qquad$ $\sum F^y = 0$

$-F_p^x + N = 0$ $\qquad\qquad$ $-3M_1 g - M_2 g + F_p^y = 0$

$\sum \tau = I\alpha$

$$+N(1m) \underbrace{- M_1 g(2m)} - M_2 g(6m) = \cancel{I\alpha}^{\,0}$$

$$|\tau| = |RF_\perp| = |RF\sin\theta| = |R_\perp F|$$

Spring + Oscillatory Motion

$$F = -kx = ma$$

$$-\frac{k}{m}x = \frac{d^2x}{dt^2} \Longrightarrow \frac{d^2x}{dt^2} = -\omega_0^2 x$$

Solution

$$\boxed{x = A\cos(\underbrace{\omega_0 t + \phi}_{})}$$

implicitly assume radian measure

$\omega_0 = \sqrt{\frac{k}{m}}$ = "angular frequency"

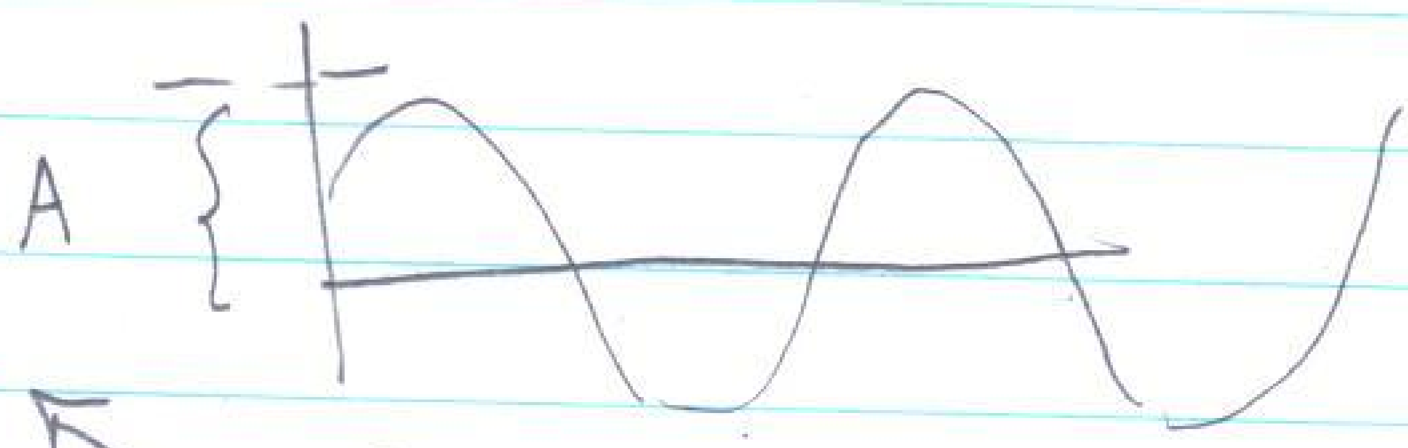
= how many radians turned per sec

$$\boxed{v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)}$$

$$\boxed{a = \frac{dv}{dt} = -\omega^2\underbrace{\left[A\cos(\omega t + \phi)\right]}_{x}}$$

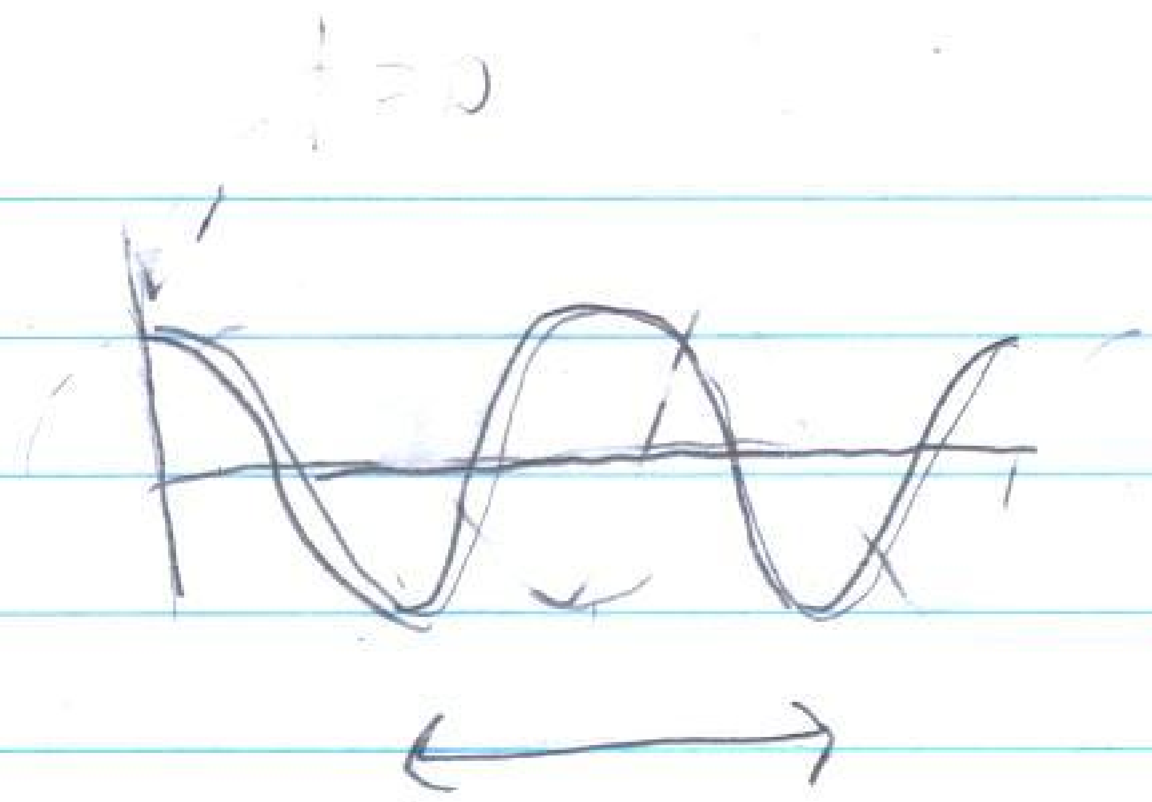
1 What are the constants

$$\boxed{x = A\cos(\omega t + \phi)}$$



Amplitude

(2)



T

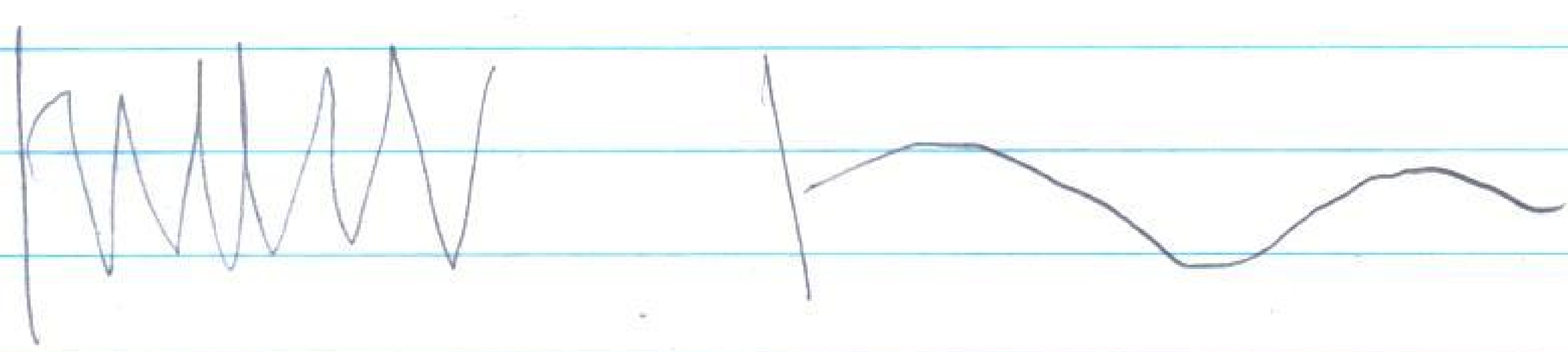
$\boxed{T = \text{time for one cycle}}$

$$(\omega(t+T)+\phi) - (\omega(t)+\phi) = 2\pi$$

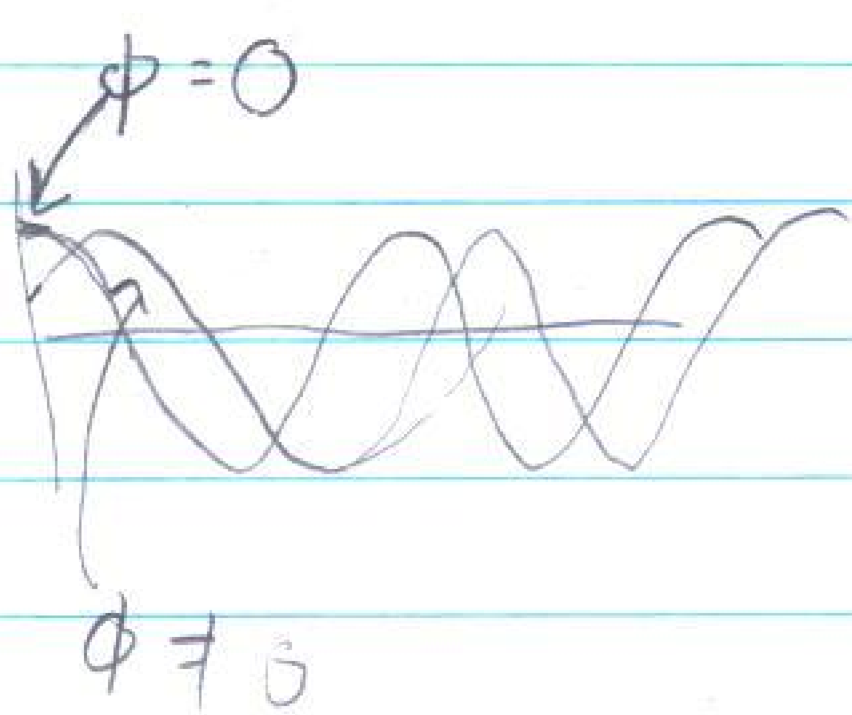
$\omega T = 2\pi$ $\quad \boxed{T = \frac{2\pi}{\omega}} \rightarrow \boxed{f = \frac{1}{T}} = \frac{\text{cycl}}{\text{se}}$

= units Hz

Example (1) ω large (2) ω small



(3) What is ϕ?



$\phi = \frac{\pi}{2}$ $\quad x = A\cos(\omega t + \frac{\pi}{2})$

$x = A\sin(\omega t)$

Example

A 200g block is connected to a light spring, force constant 5.00 N/m and oscillates on a frictionless surface. It is displaced 5.0cm from Equilibrium

a) Determine its period -- time to return

$$\omega_r = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.0\,N/m}{0.2\,kg}} = 5.0\ 1/s$$

$$T = \frac{2\pi}{\omega_r} = \frac{2\pi}{5.0\ 1/s} = 1.26\,s$$

b) Determine the Amplitude and phase

$$x = A\cos(\omega t + \phi)$$

$$V = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

we adjust A and ϕ to match the state at $t=0$, called initial condition

$$x(t=0) = 5\,cm = A\cos(\omega \cdot 0 + \phi) =$$

$$v(t=0) = 0 = -A\omega \sin(\phi)$$

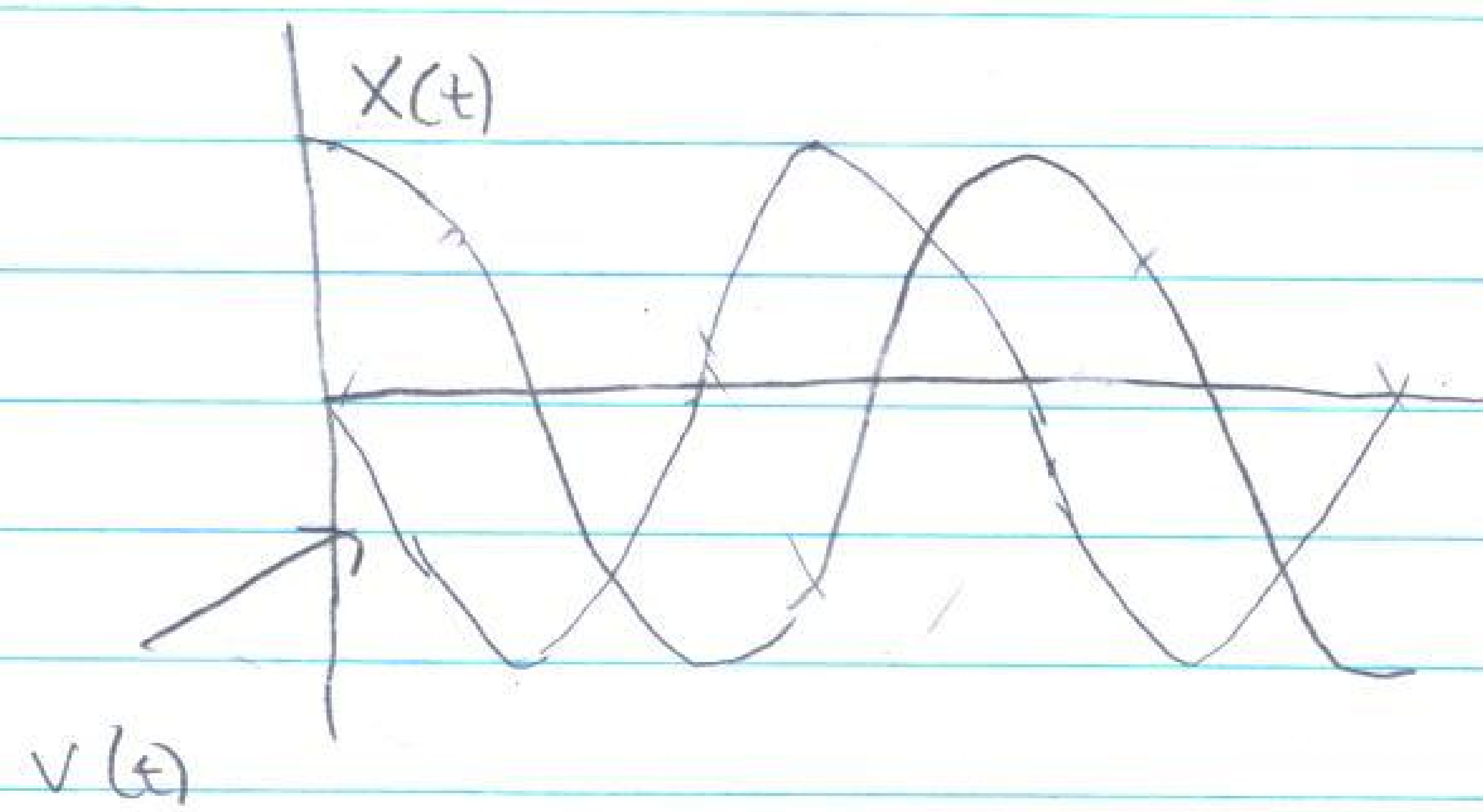
$$\phi = 0 \qquad A = 5\,cm$$

c) Find the position and velocity as a function of time

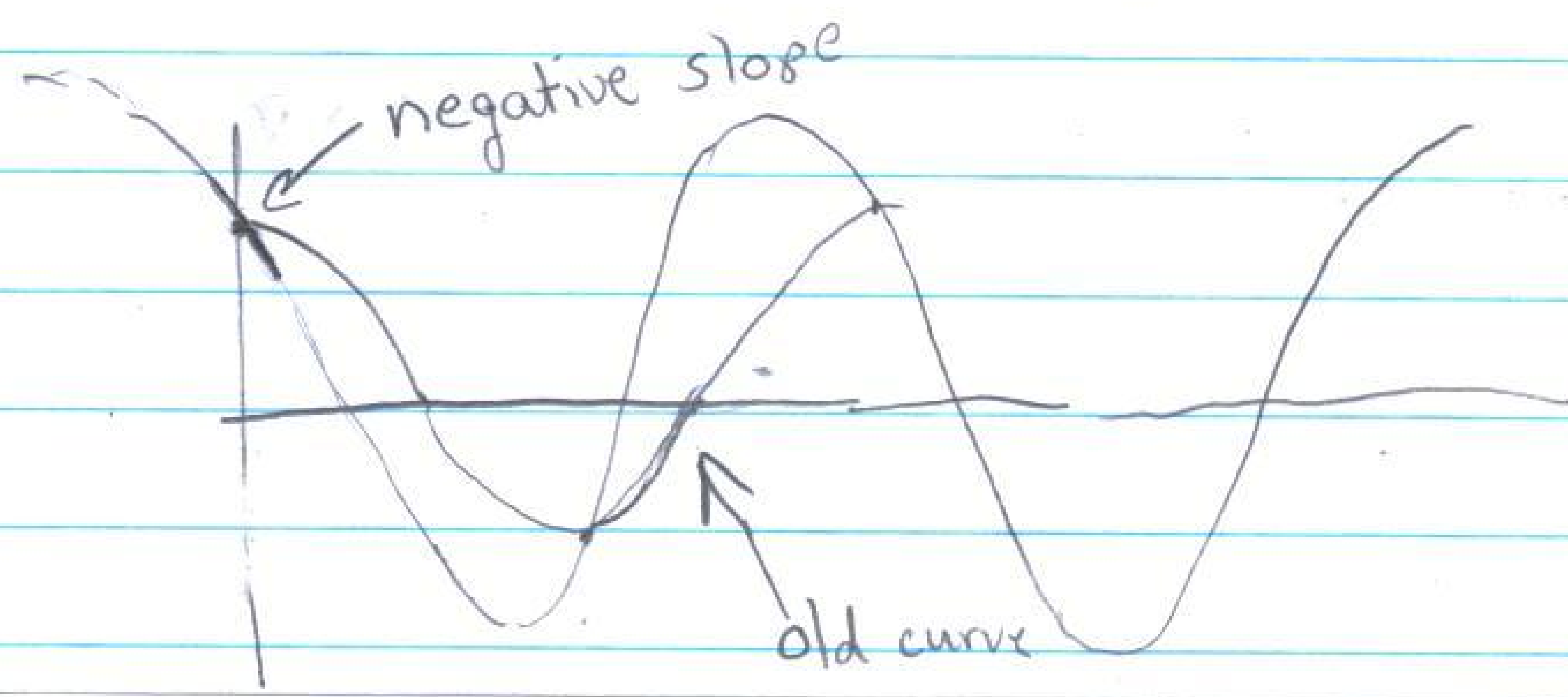
$$x = A\cos(\omega t + \phi)$$

$$x(t) = (0.05\,m)\cos\left(\frac{5.0}{s}t\right) \qquad x_{max} = 0.05\,m$$

$$v(t) = -0.25\,\frac{m}{s}\sin\left(\frac{5.0}{s}t\right) \longrightarrow v_{max} = -0.25\,m$$



d) Suppose that we give the sam block $x_i = 5\,cm$ a kick back towards equilibrium, $v_i = -.1\,m/s$



ϕ and A change but $\omega_0 = \sqrt{\frac{k}{m}}$ does not !

$$x(t) = A \cos(\omega t + \phi)$$

$$v(t) = -A\omega \sin(\omega t + \phi)$$

$$x(0) = A \cos\phi = x_i = 5\,cm$$
$$v(0) = -A\omega \sin\phi = v_i = -0.1\,m/s$$

$$\frac{-\omega \cancel{A} \sin\phi}{\cancel{A}\cos\phi} = \frac{v_i}{x_i}$$

$$\tan\phi = \frac{-v_i}{\omega x_i} = -\frac{(-0.1\,m)}{(5.0\,\frac{1}{s})(0.05\,m)} = 0.4$$

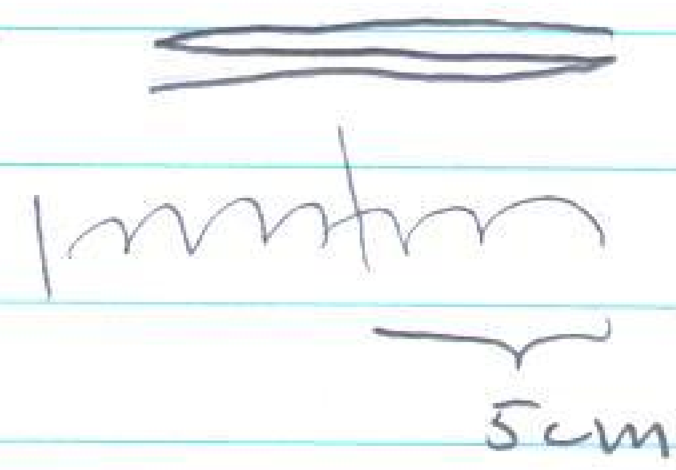
$$\phi = 0.377\ \text{rad} \qquad = 0.$$

$$A = \frac{x_i}{\cos\phi} = 0.054\,m$$

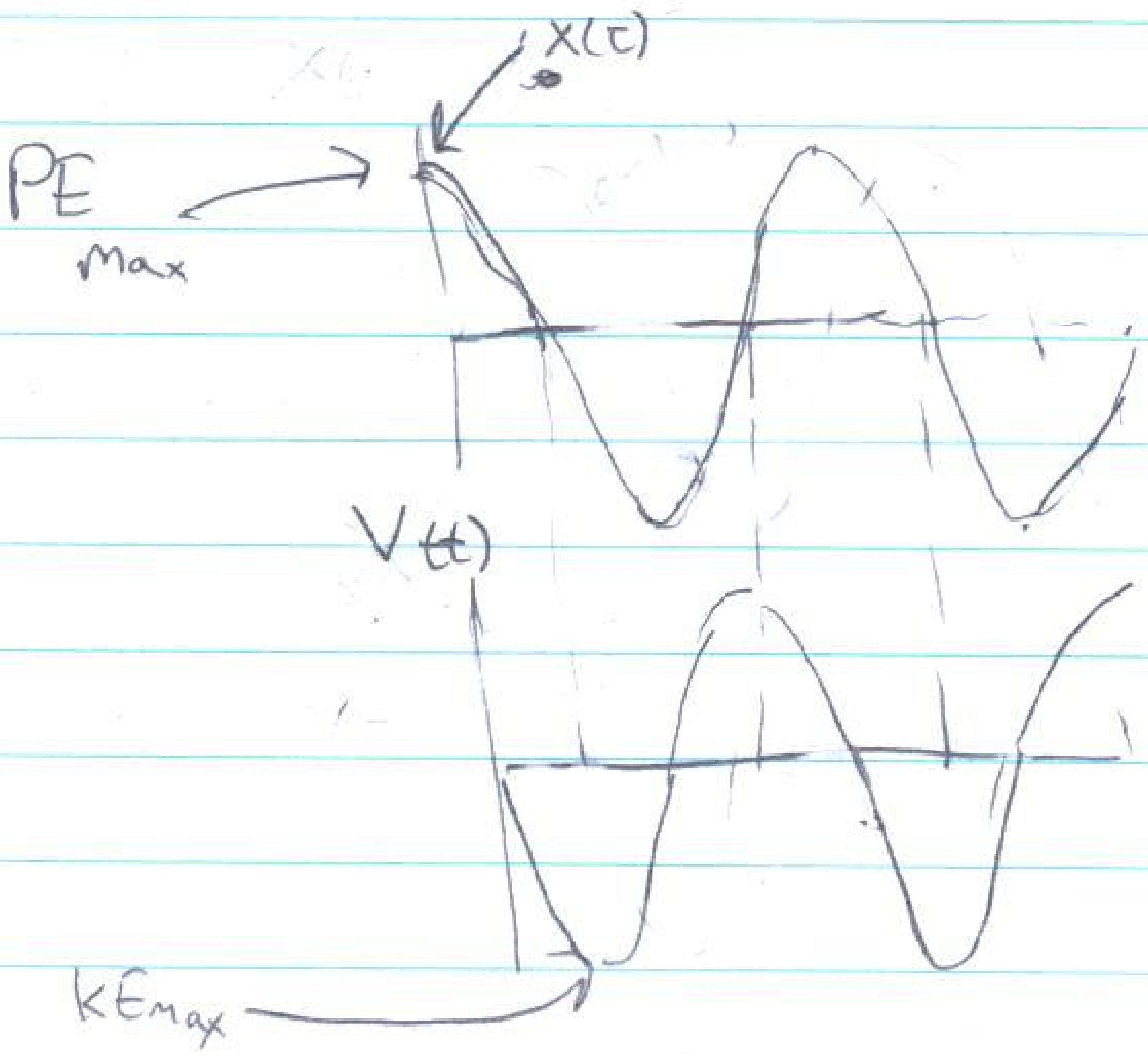
A is larger,

Itutively this is because we have given more energhy to the system by kicking it

Energy is SHM.



$$K = \frac{1}{2} m v^2(t)$$



$$x = A\cos\omega t$$

$$v = -A\omega\sin\omega t$$

Kinetic + Potential = Constant $= \frac{1}{2} k A^2$

potential energy at max extensi

Proof: $x = A\cos\omega t$

$$V = -A\omega\sin\omega t$$

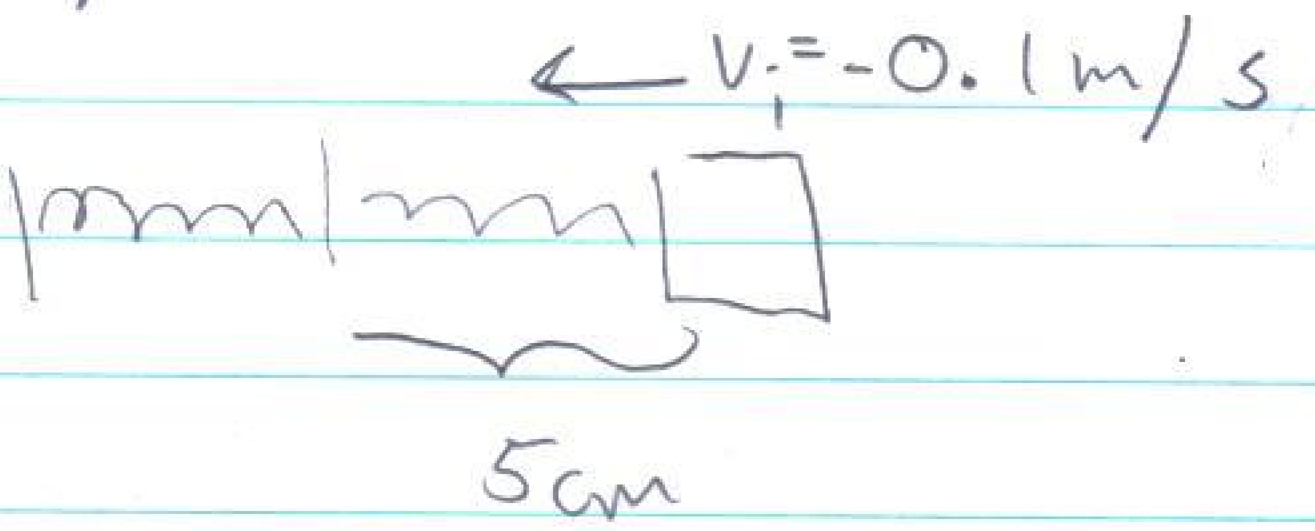
$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 =$$

$$\frac{1}{2}k\,A^2\cos^2\omega t + \frac{1}{2}m\,(-A\omega)^2\sin^2\omega$$

$$\frac{1}{2}kA^2\cos^2\omega t + \frac{1}{2}\underbrace{k\;A^2}_{m\omega^2}\sin^2\omega t$$

$$= \frac{1}{2}k^2A^2\left[\cos^2\omega t + \sin^2\omega t\right] = \frac{1}{2}kA^2$$

Problem, same as before!



Determine the amplitude of oscillation:

Sol $\underbrace{\frac{1}{2}kx_i^2 + \frac{1}{2}mv_i^2}_{\text{Energy at starting point}} = \underbrace{\frac{1}{2}kA^2}_{\text{Energy at maximum stretch is all potential } A = "x_{ma}}$

$$\sqrt{\left(x_i^2 + \frac{m}{k}v_i^2\right)} = A$$

$$\sqrt{\left(x_i^2 + \frac{v_i^2}{\omega_0^2}\right)} = A$$

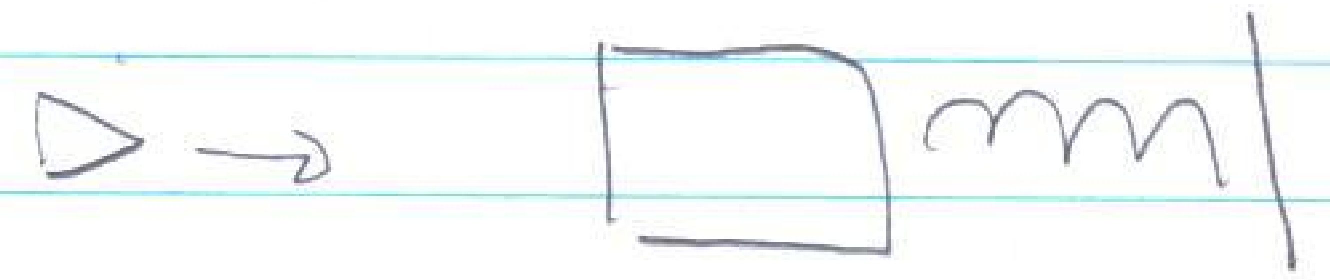
$$\sqrt{(.05\,m)^2 + \left(\frac{(-0.1\,m/s)}{(5\,1/s)}\right)^2} = A$$

$$\boxed{0.54m = A}$$ $\leftarrow$ same

Make up numbers please
I was in a rush!

Example

A bullet mass m_B, $V = V_B$ is fired into a block mass M attached to a spring w/ spring constant k



a) Find the max. compression

b) Find the equation of motion, determine the time it takes to reach $\frac{1}{4}$ of its max amplitude

a) Mom $\quad m_B v_B = (M + m_B) v_0$

$$\frac{m_B v_B}{M + m_B} = v_0$$

$$\frac{1}{2} k A^2 = \frac{1}{2} (M_B + m_B) v_0^2 = \underbrace{\frac{1}{2} m_B v_B^2}_{KE_b} \left(\frac{m_B}{M + m_B}\right)$$

$$A = \left(\frac{M + m_b}{k}\right) v_0^2 = \left(\frac{v_0}{\omega_0}\right)^2$$

b) $x_{(t)} = A \sin(\omega t) =$

$$\frac{1}{4}\cancel{A} = \cancel{A} \sin(\omega t) \longrightarrow t = \overbrace{\sin^{-1}\left(\frac{1}{4}\right)}^{\text{use rad!}} \cdot \sqrt{\frac{M + m_b}{k}}$$

Determine the maximum velocity:

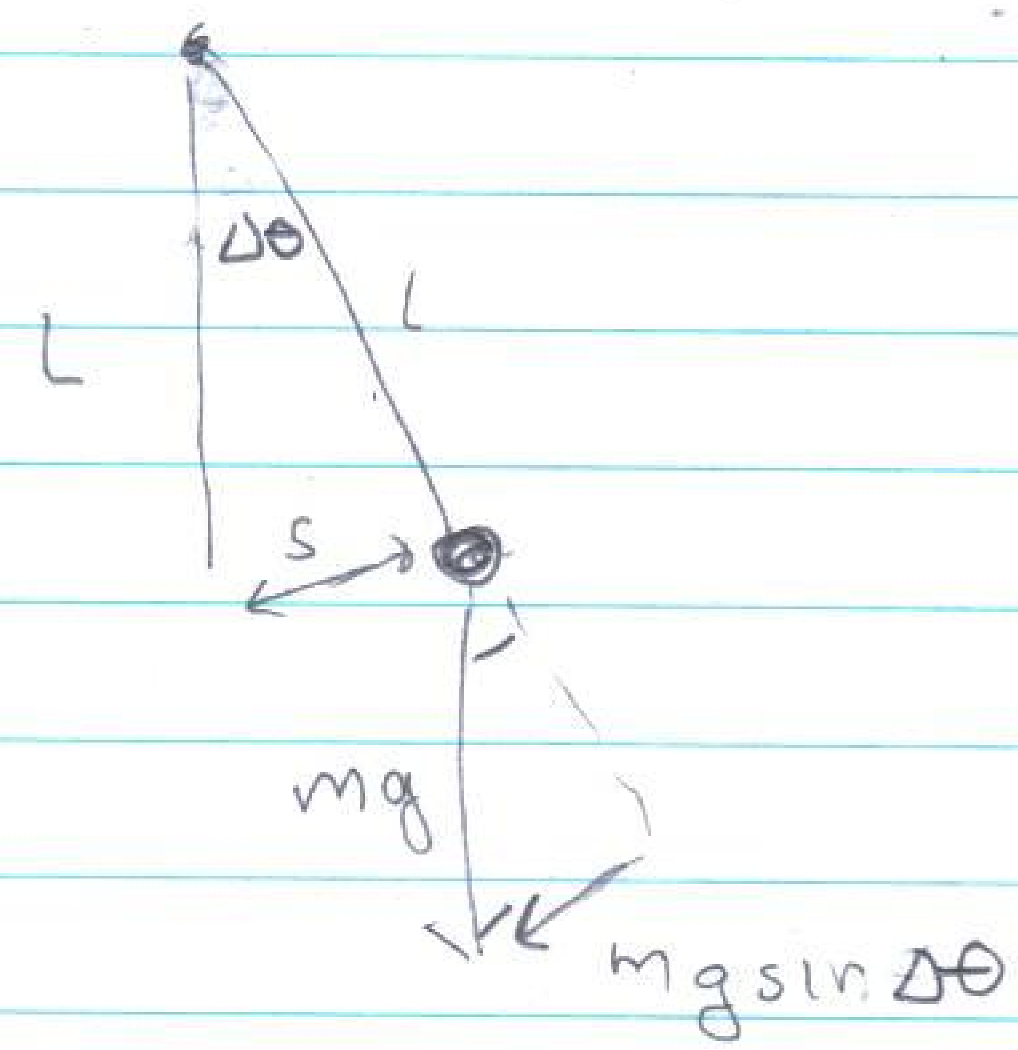
$$\frac{1}{2}kx_i^2 + \frac{1}{2}mv_i^2 = \underbrace{\frac{1}{2}mV_{max}^2}$$

Then there is no PE

$$\sqrt{\frac{k}{m}x_i^2 + v_i^2} = V_{max}$$

$$x = A\cos(\omega t + \phi)$$

Pendulum Pendulum



$$S \simeq L \, \Delta\Theta$$

$$F = ma$$

$$-mg\sin\Delta\Theta = m\frac{d^2s}{dt^2} = mL\frac{d^2(\Delta\Theta)}{dt^2}$$

$$\sin\Delta\Theta \simeq \Delta\Theta$$

$$-\cancel{m}g\,\Delta\Theta = \cancel{m}L\frac{d^2(\Delta\Theta)}{dt^2}$$

$$-\left(\frac{g}{L}\right)\Theta = \frac{d^2(\Delta\Theta)}{dt^2} \qquad \frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x$$

$$\Theta = A\cos(\omega_0 t + \phi) \qquad T = \frac{2\pi}{\omega} \text{ etc}$$

$$\omega_0 = \sqrt{\frac{g}{L}}$$

Answer Note that the result for the moment of inertia of a cylinder does not depend on L, the length of the cylinder. In other words, it applies equally well to a long cylinder and a flat disk having the same mass M and radius R. Thus, th[e] moment of inertia of the cylinder would not be affected b[y] changing its length.

Table 10.2 gives the moments of inertia for a number of objects about specific a[xes]. The moments of inertia of rigid objects with simple geometry (high symmetry) are [rel]atively easy to calculate provided the rotation axis coincides with an axis of symm[etry]. The calculation of moments of inertia about an arbitrary axis can be cumberso[me], however, even for a highly symmetric object. Fortunately, use of an important theo[rem], called the **parallel-axis theorem,** often simplifies the calculation. Suppose [the] moment of inertia about an axis through the center of mass of an object is I_{CM}. T[he] parallel-axis theorem states that the moment of inertia about any axis parallel to [and] a distance D away from this axis is

Parallel-axis theorem

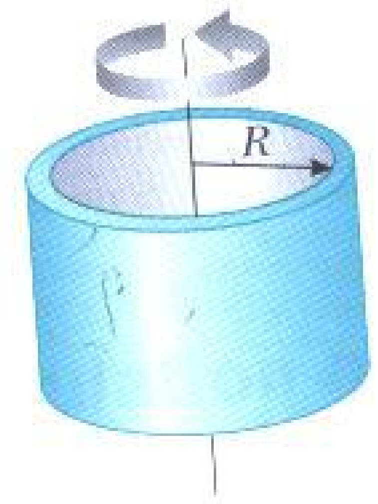
$$I = I_{CM} + MD^2$$

To prove the parallel-axis theorem, suppose that an object rotates in the xy p[lane] about the z axis, as shown in Figure 10.12, and that the coordinates of the cen[ter of] mass are x_{CM}, y_{CM}. Let the mass element dm have coordinates x, y. Becaus[e]

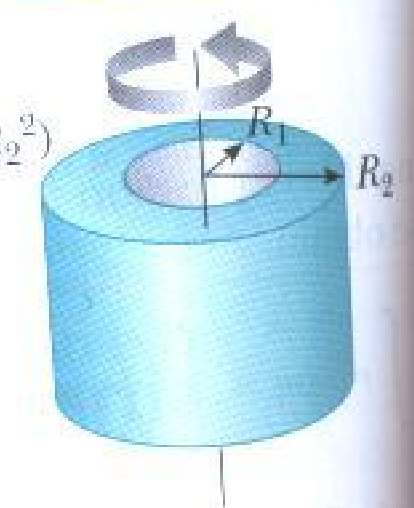
Table 10.2

Moments of Inertia of Homogeneous Rigid Objects with Different Geometries

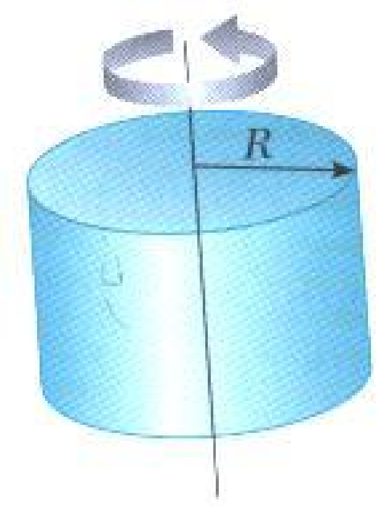
Hoop or thin cylindrical shell
$I_{CM} = MR^2$



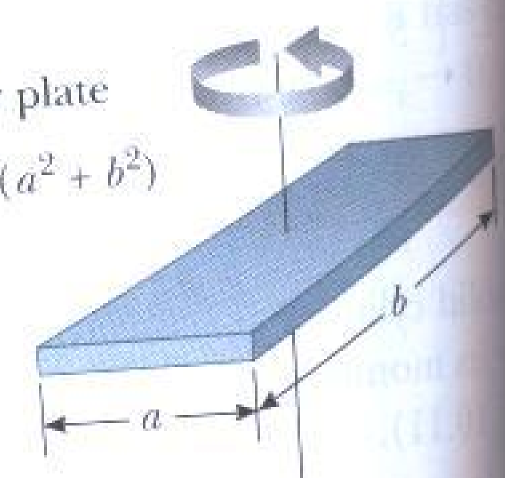
Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$



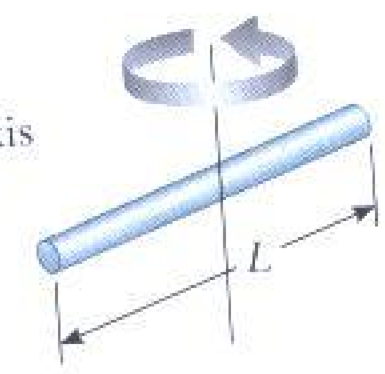
Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$



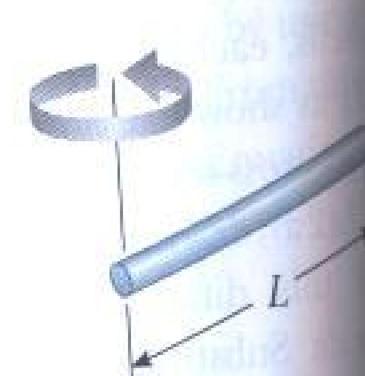
Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$



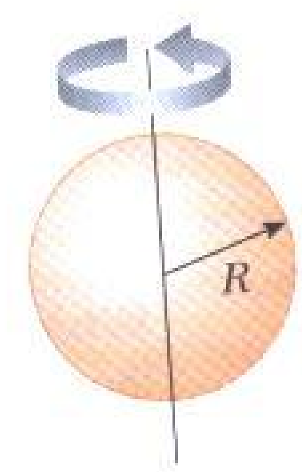
Long thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$



Long thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$



Solid sphere
$I_{CM} = \frac{2}{5}MR^2$



Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$